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Intuitionistic Fuzzy $g^{\#}$ Closed Sets

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Abstract- In this chapter the concepts of intuitionistic fuzzy g# closed set, intuitionistic fuzzy g# continuous mapping, strongly intuitionistic fuzzy g# continuous mapping, intuitionistic fuzzy g# irresolute mapping and perfectly intuitionistic fuzzy g# continuous mapping are studied. The concept of intuitionistic fuzzy g# compact is introduced. Some interesting properties are investigated besides giving some examples.

Keywords*:* intuitionistic fuzzy g# closed set, intuitionistic fuzzy g# continuous mapping, strongly intuitionistic fuzzy g# continuous mapping, intuitionistic fuzzy g# irresolute mapping, perfectly intuitionistic fuzzy g# continuous mapping, and intuitionistic fuzzy g #compact

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1. Introduction

The fuzzy concept has invaded almost all branches of mathematics ever since the introduction of fuzzy sets by L.A.Zadeh[11]. Fuzzy sets have applications in many field such as information [9] and control [10].The theory of fuzzy topological space was introduced and developed by C.L.Chang[6] and since then various notions in classical topology have been extended to fuzzy topological space. The idea of "intuitionistic fuzzy set" was first published by Atanassov[1]and many works by the same author and his colleagues appeared in the literature [2-4].Later this concept was generalized to "intuitionistic L -fuzzy set" by Atanassov and stoeva[5]. The concept of generalized intuitionistic fuzzy closed set was introduced by R.Dhavaseelan , E.Roja and M.K.Uma [7].The concept of intuitionistic fuzzy generalized alpha open set was introduced by D.Kalamani , K.Sakthivel and C.S.Gowri [8]. In this chapter the concepts of intuitionistic fuzzy g# closed set, intuitionistic fuzzy g# continuous mapping, strongly intuitionistic fuzzy g# continuous mapping, intuitionistic fuzzy g# irresolute mapping and perfectly intuitionistic fuzzy g# continuous mapping are studied. The concept of intuitionistic fuzzy g# compact is introduced. Some interesting properties are investigated besides giving some examples.

2. Preliminaries

Definition 2.1[3]: Let X be a nonempty fixed set. An intuitionistic fuzzy set(IFS for short)A is an object having the form A = $\{< x, \mu_A(x), \delta_A(x) >: x \in X\}$ where the function μ_A : X→1 and $\delta_A(x)$: X→1 denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmember ship $(\delta_A(x))$of each element $x \in X$ to the set A respectively ,and 0≤ $\mu_A(x) + \delta_A(x)$ ≤1 for each $x \in X$.

Definition 2.2[3]: Let X be an nonempty set and intuitionistic fuzzy sets A and B in the form

A = $\{< x, \mu_A(x), \delta_A(x) >: x \in X\}$, B = $\{< x, \mu_B(x), \delta_B(x) >: x \in X\}$ .Then

(a) $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x)$ and $\delta_A(x) \geq \delta_B(x)$ for all $x \in X$;

(b) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;

(c) $\bar{A} = \{< x, \delta_A(x), \mu_A(x) >: x \in X\}$

(d) $A \cap B = \{< x, \mu_A(x) \cap \mu_B(x), \delta_A(x) \cup \delta_B(x) >: x \in X\}$

(e) $A \cup B = \{< x, \mu_A(x) \cup \mu_B(x), \delta_A(x) \cap \delta_B(x) >: x \in X\}$

(f) $A = \{< x, \mu_A(x), 1 - \mu_A(x) >: x \in X\}$

(g)[ ] $A = \{< x, 1 - \delta_A(x), \delta_A(x) >: x \in X\}$

Definition 2.3[3]: Let $\{A_i : i \in J\}$be an arbitrary family of intuitionistic fuzzy sets in X. Then

(a) $\cap A_i = \{< x, \cap \mu_{A_i}(x), \cup \delta_{A_i}(x) >: x \in X\}$

(b) $\cup A_i = \{< x, \cup \mu_{A_i}(x), \cap \delta_{A_i}(x) >: x \in X\}$

Since our main purpose is to construct the tools for developing intuitionistic fuzzy topological spaces , we must introduce the intuitionistic fuzzy sets $0_\sim$ and $1_\sim$ in X as follows:

Definition 2.4[3]:

(a) $0_\sim = \{< x, 0, 1 >: x \in X\}$

(b) $1_\sim = \{< x, 1, 0 >: x \in X\}$

Definition 2.5[3]: An intuitionistic fuzzy topology on X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i \setminus i \in J\}$ in τ

In this case the pair (X, τ) is called an intuitionstic fuzzy topology space and any intuitionstic fuzzy set in τ is known as an intuitionistic fuzzy open set in X.

The complement A^c of an intuitionistic fuzzy open set A in an intuitionstic fuzzy topology space (X, τ) is called an intuitionistic fuzzy closed set in X.

Definition 2.6[3]:(a) If B = $\{< y, \mu_B(x), \delta_B(x) >: y \in Y\}$ is an intuitionistic fuzzy set in Y, then the preimage of B under f , denoted by $f^{-1}(B)$, is the intuitionistic fuzzy set in X defined by $f^{-1}(B) = \{< x, f^{-1}(\mu_B)(x), f^{-1}(\delta_B)(x) >: x \in X\}$

(b) If A = $\{< x, \lambda_A(x), \gamma_A(x) >: x \in X\}$ is an intuitionistic fuzzy set in X, then the image of A under f , denoted by $f(A)$, is the intuitionistic fuzzy set in Y defined by $f(A) = \{< y, f(\lambda_A)(y), (1 - f(1 - \gamma_A))(y) >: y \in Y\}$. Where

$$f(\lambda_A)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \lambda_A(x), & if \ f^{-1}(y) \neq 0 \\ 0, & otherwise \end{cases}$$

$$(1 - f(1 - \gamma_A))(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \gamma_A(x), & if \ f^{-1}(y) \neq 0 \\ 1, & otherwise \end{cases}$$

for the sake of simplicity , let us use the symbol $f\_(\gamma_A)$ for $(1 - f(1 - \gamma_A))$.

Definition 2.8[7]: Let (X,T) be an intuitionistic fuzzy topological space . An intuitionistic fuzzy set A in (X,T) is said to be generalized intuitionistic fuzzy closed if intuitionistic fuzzy cl(A)⊆ G whenever A⊆ G and G is intuitionistic fuzzy open.The complement of a generalized intuitionistic fuzzy closedset is generalized intuitionistic fuzzy open.

Definition 2.9[8]: An intuitionistic fuzzy set A in(X, τ) is said to be an intuitionistic fuzzy generalized alpha open set if $\propto \operatorname{int}(A) \supseteq U$ whenever A⊇U and U is an intuitionistic fuzzy alpha closed set in (X, τ)

3. Intuitionistic fuzzy g# Closed sets and Intuitionistic fuzzy g# continuous functions

Definition 3.1: Let (X, T) be an intuitionistic fuzzy topological space and Let A be an intuitionistic fuzzy set in X. Then A is said to be intuitionistic fuzzy g# closed if IFcl (A) ⊆U whenever A⊆U and U is an intuitionistic fuzzy generalized α open.

The complement of intuitionistic fuzzy g# closed set is intuitionistic fuzzy g# open.

Definition 3.2: Let (X, T) be an intuitionistic fuzzy topological space and Let A be an intuitionistic fuzzy set in X .Then intuitionistic fuzzy g# closure and intuitionistic fuzzy g# interior are defined by

(a) IFg#cl (A) = ∩{G\ G is a IF g# closed set in X and A⊆G}

(b) IFg#int (A) = ∪ {K\ K is a IF g# open set in X and K⊆A}

Definition 3.3: Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces.

(a) A map f :(X,T)→(Y,S) is said to be intuitionistic fuzzy g# continuous if the pre image of every open set in (Y,S) is intuitionistic fuzzy g# open set in (X,T).

(b) A map f :(X,T)→(Y,S) is said to be intuitionistic fuzzy g# irresolute if the preimage of every intuitionistic fuzzy g# open set in (Y,S) is intuitionistic fuzzy g# open set in (X,T).

(c) A map f :(X,T)→(Y,S) is said to be strongly intuitionistic fuzzy g# continuous if the preimage of every intuitionistic fuzzy g# open set in (Y,S) is intuitionistic fuzzy open set in (X,T).

(d) A map f :(X,T)→(Y,S) is said to be perfectly intuitionistic fuzzy g# continuous if the preimage of every intuitionistic fuzzy g# open set in (Y,S) is both intuitionistic fuzzy open and intuitionistic fuzzy closed in (X,T).

Proposition 3.4: Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let f :(X,T)→(Y,S) be intuitionstic fuzzy g# continuous .Then for every intuitionstic fuzzy set A in X , f(IFg#cl(A)) ⊆ IFcl(f(A))

Proof: Let A be an intuitionistic fuzzy set in (X, T). Since IFcl (f (A)) is an intuitionistic fuzzy closed set and f is a intuitionstic fuzzy g# continuous mapping, f^{-1}(IFcl(f(A)) is a intuitionstic fuzzy g# closed set and f^{-1}(IFcl(f(A)) ⊒ A .Now IFg#cl(A) ⊆ f^{-1}(IFcl(f(A). Therefore f (IFg#cl(A)) ⊆ IFcl(f(A)).

Proposition 3.5: Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. Let f :(X,T)→(Y,S) be intuitionstic fuzzy g# continuous .Then for every intuitionstic fuzzy set A in Y, IFg#cl(f^{-1}(A)) ⊆ f^{-1} (IFcl(A))

Proof: Let A be an intuitionistic fuzzy set in (Y,S). Let B = f^{-1}(A).Then

f(B) = f(f^{-1}(A) ⊆ A. By proposition 3.4, f(IFg#cl(f^{-1}(A))) ⊆ IFcl(f(f^{-1}(A)) .

Thus IFg#cl(f^{-1}(A)) ⊆ f^{-1}(IFcl(A)).

Proposition 3.6: Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces. If A is a IFg# closed set in (X,T) and if f :(X,T)→(Y,S) is intuitionstic fuzzy continuous and intuitionstic fuzzy closed, then f(A) is IFg# closed in(Y,S).

Proposition 3.7: Let (X, T) and (Y,S) be any two intuitionistic fuzzy topological spaces. if f :(X,T)→(Y,S) is intuitionstic fuzzy continuous then it is IFg# continuous.

The converse of Proposition 3.7 need not true. See Example 3.8

Example 3.8: Let X= {a,b,c}and Y={a,b,c} Define the intuitionstic fuzzy sets A,B and C as follows

A = $\langle x, (0.4,0.4,0.4), (0.5,0.5,0.5)\rangle$

B = $\langle x, (0.5,0.6,0.7), (0.4,0.3,0.2)\rangle$ and

C = $\langle x, (0.4,0.6,0.5), (0.3,0.3,0.4)\rangle$.

Then T = {$0_\sim$, $1_\sim$, A, B} and S = {$0_\sim$,$1_\sim$,C} are intuitionstic fuzzy topologies on X and Y. Thus (X, T) and (Y, S) are intuitionistic fuzzy topological spaces. Define f :(X,T)→(Y,S) as follows . f(a)=a,f(b)=b,f(c)=c. Clearly f is IFg# continuous. Now f is not IF continuous, Since for f^{-1} (C) ∉ Tfor C∈ S.

Propositions 3.9: Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. If f :(X,T)→(Y,S) is intuitionstic fuzzy g# irresolute then it is IFg# continuous.

The converse of Proposition need not true. See Example 3.10

Example 3.10: Let X= {a,b,c}and Y={a,b,c}. Define the intuitionistic fuzzy sets A, B and C as follows

A = $\langle x, (0.4,0.5,0.5), (0.5,0.5,0.5)\rangle$

B = $\langle x, (0.6,0.7,0.6), (0.4,0.3,0.4)\rangle$ and

C = $\langle x, (0.5,0.5,0.5), (0.4,0.4,0.4)\rangle$.

Then T = {$0_\sim$, $1_\sim$, A, B} and S = {$0_\sim$,$1_\sim$,C} are intuitionstic fuzzy topologies on X and Y. Thus (X, T) and (Y, S) are intuitionstic fuzzy topological spaces. Define f :(X, T)→(Y,S) as follows . f (a)=a,f(b)=b,f(c)=c. Clearly f is IFg# continuous. But f is not IF g# irresolute. Since D = $\langle x, (0.6,0.6,0.5), (0.4,0.4,0.5) \rangle$ is IFg# open in (Y,S), f^{-1}(D) is not IFg# open in (X,T)

Proposition 3.11: Let (X, T) and (Y, S) be any two intuitionistic fuzzy topological spaces. if f :(X,T)→(Y,S) is strongly intuitionistic fuzzy g# continuous then it is intuitionistic fuzzy continuous.

The converse of Proposition need not true. See Example 3.12

Example 3.12: Let X= {a,b,c}and Y={a,b,c}. Define the intuitionistic fuzzy sets A, B and C as follows

A = $\langle x, (0.5,0.4,0.5), (0.3,0.3,0.3) \rangle$

B = $\langle x, (0.6,0.5,0.6), (0.3,0.2,0.3) \rangle$ and

C = $\langle x, (0.5,0.5,0.6), (0.2,0.2,0.3) \rangle$.

ThenT = {$0_\sim$,$1_\sim$,A,B} and S = {$0_\sim$,$1_\sim$,C} are intuitionstic fuzzy topologies on X and Y. Thus (X,T) and (Y,S) are intuitionstic fuzzy topological spaces. Define f :(X,T)→(Y,S) as follows . f(a)=c,f(b)=b,f(c)=c. Clearly f is IF continuous. But f is not strongly IF g# irresolute. Since D = $\langle x, (0.7,0.8,0.7), (0.1,0.1,0.1) \rangle$ is IFg# open in (Y, S), f^{-1}(D) is not IF open in (X,T)

Proposition 3.13: Let (X, T) and (Y,S) be any two intuitionistic fuzzy topological spaces. if f :(X,T)→(Y,S) is perfectly intuitionstic fuzzy g# continuous then it is strongly intuitionstic fuzzy g# continuous.

The converse of Proposition need not true. See Example 3.14

Example 3.14: Let X= {a,b,c}, Y={a,b,c} .Define the intuitionstic fuzzy sets A,B and C as follows

A = $\langle x, (0.8,0.8,0.8), (0.1,0.1,0.1) \rangle$

B = $\langle x, (0.6,0.5,0.6), (0.3,0.2,0.3) \rangle$ and

C = $\langle x, (0.5,0.5,0.6), (0.2,0.2,0.3) \rangle$. Then

T = {$0_\sim$,$1_\sim$,A,B} , S = {$0_\sim$,$1_\sim$,B} are intuitionstic fuzzy topologies on X and Y . Thus (X,T) and (Y,S) are intuitionstic fuzzy topological spaces. Define f :(X,T)→(Y,S) as follows . f(a)=c, f(b)=b, f(c)=c . Clearly f is strongly intuitionstic fuzzy g# continuous. But f is not perfectly intuitionstic fuzzy g# continuous.

Since D = $\langle x, (0.8,0.8,0.8), (0.1,0.1,0.1) \rangle$ is IFg# open in (Y, S), f^{-1}(D) is intuitionistic fuzzy open and not intuitionistic fuzzy closed in (X,T)

Proposition 3.15: Let (X,T) , (Y,S)and (Z,R) be any three intuitionistic fuzzy topological spaces. Suppose f :(X,T)→(Y,S) , g :(Y,S)→(Z,R)be maps .Assume f is intuitionstic fuzzy g# irresolute and g is intuitionstic fuzzy g# continuous then g o f is intuitionstic fuzzy g# continuous.

Proposition 3.16: Let (X,T) , (Y,S)and (Z,R) be any three intuitionistic fuzzy topological spaces. Suppose f :(X,T)→(Y,S) , g :(Y,S)→(Z,R)be maps .Assume f is strongly intuitionstic fuzzy g# continuous and g is intuitionstic fuzzy g# continuous then g o f is intuitionstic fuzzy continuous.

Definition 3.17: An intuitionistic fuzzy topological space (X,T) is said to be intuitionistic fuzzy $T_{1/2}$ if every intuitionistic fuzzy g# closed set in (X,T) is intuitionistic fuzzy closed in (X,T).

Proposition 3.18: Let (X,T) , (Y,S)and (Z,R) be any three intuitionistic fuzzy topological spaces.Let f :(X,T)→(Y,S) & g :(Y,S)→(Z,R)be maps and (Y,S) be intuitionistic fuzzy $T_{1/2}$ .If f and g are intuitionstic fuzzy g# continuous then g o f is intuitionstic fuzzy g# continuous.
This proposition is not valid if (Y,S) is not intuitionistic fuzzy $T_{1/2}$.

Example 3.19: Let X= {a,b,c}. Define the intuitionistic fuzzy sets A,B and C as follows.

A = $\langle x, (0.4,0.4,0.6), (0.4,0.4,0.3)\rangle$

B = $\langle x, (0.5,0.5,0.6), (0.3,0.4,0.3)\rangle$ and

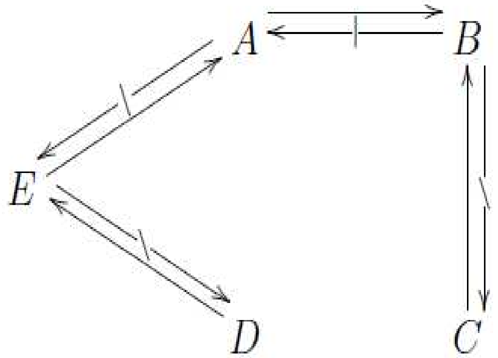
C = $\langle x, (0.6,0.6,0.6), (0.5,0.3,0.4)\rangle$

Then the family T = {0,1~,A},S= {0,1~,B} and R = {0,1~,C} are intuitionistic fuzzy topologies on X.Thus (X,T),(X,S)and(X,R) are intuitionistic fuzzy topology spaces on X. Also define f :(X,T)→(Y,S) as f(a)=a, f(b)=b &f(c)=c and g :(Y,S)→(Z,R) as g(a)=a, g(b)=b &g(c)=c. clearly f and g are intuitionstic fuzzy g# continuous. But g o f is not intuitionstic fuzzy g# continuous. Since $\overline{C}$ is intuitionistic fuzzy closed in (X,R). therefore $f^{-1}(g^{-1}(\overline{C}))$ is not intuitionistic fuzzy closed in (X,T) because (X,S) is not intuitionistic fuzzy $T_{1/2}$ space. Therefore g o f is not intuitionstic fuzzy g# continuous.

INTERRELATION

From the above results proved,we have a diagram of implications as shown below.

In the diagram A,B,C,D and E denote intuitionistic fuzzy continuity,IF $g^{\#}$-continuity, intuitionistic fuzzy $g^{\#}$ irresolute, perfectly IF $g^{\#}$-continuity and strongly IF $g^{\#}$-continuity respectively.



4. Intuitionistic fuzzy g# compactness

Definition 4.1: Let (X,T) be an IFTS, if a family $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ of intuitionistic fuzzy g# open spaces in X satisfies the condition $\cup \{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ = 1~ . Then it is called a intuitionistic fuzzy g# open cover of X.
A finite sub family of a intuitionistic fuzzy g# open cover $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ of X which is also a intuitionistic fuzzy g# open cover of X , is called a finite sub cover of $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$

Definition 4.2: An Intuitionistic fuzzy topological space (X,T) is called Intuitionistic fuzzy g# compact iff every Intuitionistic fuzzy g# open cover of X has a finite sub cover.

Proposition 4.3: Let (X,T) and (Y,S) be any intuitionistic fuzzy topological spaces and f: (X,T) → (Y,S) be intuitionistic fuzzy g# continuous surjection. If (X,T) is intuitionistic fuzzy g# compact then (Y,S) is also intuitionistic fuzzy g# compact.

Definition 4.4:
Let (X, T) be an intuitionistic fuzzy topological space and A be an intuitionistic fuzzy set in (X,T).If a family $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ of intuitionistic fuzzy g# open sets in (X, T) satisfies the condition A $\subseteq$ $\cup$ $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ = $1_\sim$ then it is called an intuitionistic fuzzy g# open cover of A.A finite sub family of a intuitionistic fuzzy g# open cover $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$ of A , which is also a intuitionistic fuzzy g# open cover of A , is called a finite sub cover of $\{< x, \mu_{G_i}, \gamma_{G_i} >; i \in J\}$

Definition 4.5:
An intuitionistic fuzzy set A is called intuitionistic fuzzy g# compact iff every intuitionistic fuzzy g# open cover of A has a finite sub cover.

Proposition 4.6:
Let (X,T) and (Y,S) be any two intuitionistic fuzzy topological spaces and f: (X,T) → (Y,S) be intuitionistic fuzzy g# continuous function. If A is intuitionistic fuzzy g# compact in (X,T) , then is f(A) is intuitionistic fuzzy g# compact in (Y,S).

Proposition 4.7:
Let (X,T) be an intuitionistic fuzzy g# compact space and suppose that A is a intuitionistic fuzzy g# closed set of (X,T)then A is an intuitionistic fuzzy g# compact .

References

[1] Atanassov.K Intuitionistic fuzzy sets in V.Sgurev.Ed., VII ITKR's session, Sofia.(1984).

[2] Atanassov.K Intuitionistic fuzzy sets . *Fuzzy sets and systems*,20.pp.87-96. (1986)

[3] Atanassov.K Review and new results on Intuitionistic fuzzy sets, *Preprint IM-MFAIS*-1-88,Sofia.(1988)

[4]Atanassov.K and Stoeva.S. Intuitionistic fuzzy sets, in *: PolishSyrup. on interval & Fuzzy Mathematics*, Poznan, pp.23-36. (August)(1983.)

[5] Atanassov.K and Stoeva.S. Intuitionistic L- fuzzy sets, in: *.Trappal,Ed., Cybernetics and system Research*,Vol .2 (Elsevier,Amsterdam).pp.539-540 (1984).

[6] Chang.C.L.Fuzzy topological Spaces.*J.Math.Anal.Appl.*,24,pp.182-190. (1968)

[7] Dhavaseelan.R,Roja.E andUma.M.K. Generalized intuitionistic fuzzy closed sets .*Advances in Fuzzy Mathematics*, Volume 5,Number 2 ,pp.157-172. (2010)

[8] Kalamani.D,Sakthivel.K and Gowri. C.S. Generalized Alpha Closed Sets in Intuitionistic Fuzzy Topological Spaces. *Applied mathematical Sciences* , Volume 6 , Number 94, 4691-4700.

[9] Smets.P.The degree of belief in a fuzzy event, *Information Sciences*,25 pp.1-19. (1981)

[10] Sugeno. M.An Introductory survey of fuzzy control, Information Sciences,36,pp.59-83. (1985)

[11] Zadeh. L.A.Fuzzy sets,Informand control , 8.pp.338-353.